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Romero, Ric A.

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Kevin D. Shepherd, Ric A. Romero
Department of Electrical and Computer Engineering
Naval Postgraduate School
Monterey, CA, USA
{kdshephe,rnromero}@nps.edu

***Abstract*—In this paper, we investigate spectrally adaptive radar transmit waveform design and its effects on an active communication system. We specifically look at waveform design for point targets. The transmit waveform is optimized by accounting for the modulation spectrum of the communication system while trying to efficiently use the remaining spectrum. With the use of spectrally-matched radar waveform, we show that the SER detection performance of the communication system is minimally affected compared to the SER performance with a classical non-adaptive pulsed radar waveform. Moreover, we show the detection performance of the adaptive waveform is less impacted by the active communication compared to that of the pulsed radar waveform design. In other words the radar is able to coexist with a friendly communication system and thus share the spectrum with a friendly communication system.**

***Index Terms*—waveform design, spectrum sharing, cognitive radar, electronic warfare, spectrum management**

I. Introduction

One of the primary problems of electronic warfare (EW) is the issue of spectral crowding between friendly users on top of the non-friendly users. As the battlefield grows more and more crowded with RF transmitters, it has become increasingly apparent that new systems are going to have to account for this crowding and attempt to manage the overall spectral environment. It is theorized that on the modern battlefield, with numerous transmitters and jammers present, we may find a need to use the same portion of the spectrum that is assigned to friendly systems. Indeed, in practice total disruption to friendly radio systems is encountered when a high-powered radar system is turned on. It is not only important that the radar be able to detect targets in the presence of the legacy friendly communication signal, but also that it not be disruptive to that same communications signal. In this paper, we consider the design of transmit radar waveform for point targets such that it mitigates the interference effects of an active friendly communication system and vice versa.

The primary difference between traditional radar, one with a pre-determined transmit pulse shape, and a cognitive radar is that the cognitive radar looks at the preceding returns and uses the information present to make a decision as to what the next transmit pulse should look like. In this manner the cognitive radar makes the best use of the spectrum at its disposal. This is slightly similar to the concept of manual frequency hopping, where a receiver monitors the spectrum and places a pulse in the least noisy segment of the spectrum. However, for a cognitive radar with the ability to respond to dynamic spectrum changes, there is a continuous reshaping of the pulse to minimize the use of noisy and interfered bands. The transmit waveform method described here can be used for cognitive radar.

In [1] transmit pulse shaping based on maximization of both signal-to-noise ratio (SNR) and mutual information (MI) for signal-dependent interference is investigated. A similar method, applied to point targets and labeled as 'waterfiling' method for spectrum shaping, is further discussed in [2]. In [3], generalized match filter design for signal detection which will be used here for receive processing is discussed. In this paper, these methods [1-3] are used as starting points for waveform design and detection.

In section II, we show a method for designing a waveform for a point target in the presence of a friendly communication signal in the radar's received spectrum. In section III we show the improved performance of the radar receiver due to the adaptive transmit waveform despite the friendly communication signal interference as compared to a traditional pulsed transmit signal. In section IV we show the benefit of this waveform on the existing communication system i.e. the performance of the communication system is minimally affected compared to the the performance degradation if a traditional wideband pulse is used. In section V we present the results when multiple communications systems are present within the working spectrum of the radar. Finally in section VI we present our conclusions.

II. Waveform design

When a target is not present, the received signal prior to transmission is represented by

$$x(t) = q(t) + n(t) \tag{1}$$

where $q(t)$ is the time domain representation of a friendly communication signal and $n(t)$ represents the AWGN noise out of the receiver. In the frequency domain this becomes

$$X(f) = Q(f) + N(f). \tag{2}$$

In order to find an effective transmit signal design, we use the SNR-based method derived in [1] and [2] known as the 'waterfilling' technique. This technique examines the

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interference spectrum and places the energy of the transmitted signal where the total interference is spectrally low. To execute this technique we start with the energy constraint of the radar transmitter, which is the amount of energy that the radar can placed in one transmit pulse and is given by

$$\epsilon = \int_{-w/2}^{w/2} T|S(f)|^2 df \tag{3}$$

where T is the duration of the transmit pulse and $S(f)$ is the time-normalized Fourier transform of the transmit waveform $s(t)$. Of course, the equivalent power constraint is nothing but energy divided by the time support. We then waterfill the spectrum as dictated by

$$\epsilon_s(f) = T|S(f)|^2 = \max\left(\frac{\sqrt{P_x(f)/\lambda} - P_x(f)}{P_h(f)}, 0\right) \tag{4}$$

where $P_h(f)$ represents the PSD of the clutter response which is sometimes present (e.g. ground-looking radar), $P_x(f)$ represents the PSD of the interference plus noise, and we solve for λ that satisfies the energy constraint of the radar transmitter. Next we divide by T to arrive at the optimal transmit power spectral density (PSD) $|S(f)|^2$ in the frequency domain.

Consider a friendly communications system using a quartenary phase-shift keying (QPSK) modulation in which the possible spectra (depending on the bandwidth used) are shown in the top panel of Fig. 1 and 2. Via (4), we can calculate the optimum transmit spectrum. The corresponding optimal spectra of the transmit pulse are shown in the bottom panel of Fig. 1 and 2. Clearly there could be plenty of waveforms that fit one of the spectra since an optimal spectrum is phase tolerant. For the simulations in this work, it is sufficient to chose one realization of the many possible ones. Furthermore, we see the difference in the two transmit spectra based on the amount of bandwidth available to the radar pulse.

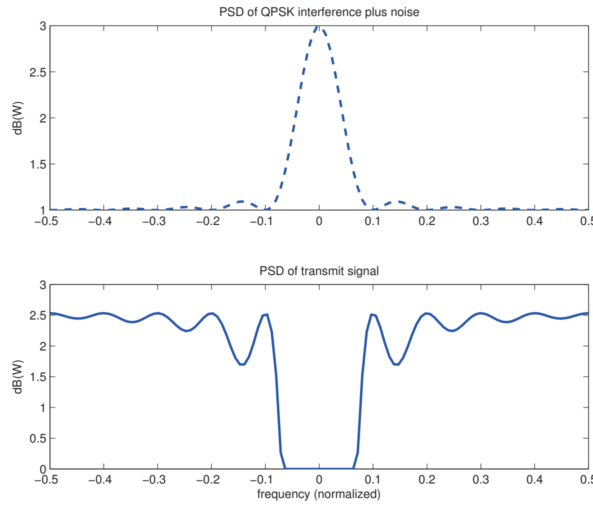

Fig. 1. Top Panel: PSD of QPSK with smaller bandwidth than radar signal. Bottom Panel: PSD of resulting spectrum of the radar transmit signal.

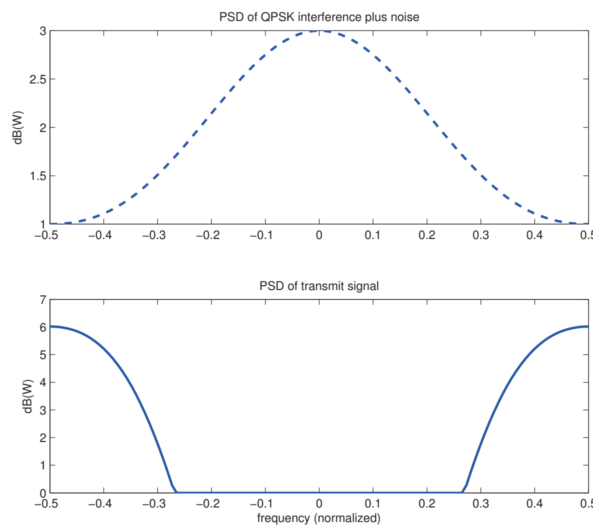

Fig. 2. Top Panel: PSD of QPSK with equally allocated bandwidth to the radar signal. Bottom Panel: PSD of resulting spectrum of the radar transmit signal.

III. Radar Receiver

Now given a realization of the transmit waveform we form the two detection hypotheses given by

$$\mathcal{H}_0 : x(t) = s(t) * h(t) + q(t) + n(t) \tag{5}$$

$$\mathcal{H}_1 : x(t) = As(t) + s(t) * h(t) + q(t) + n(t) \tag{6}$$

where $x(t)$ is the received signal, $s(t)$ is the transmitted signal, $As(t)$ is the deterministic radar response of a point target with amplitude A, $h(t)$ is the clutter response, the convolution $s(t) * h(t)$ is the clutter echo, $q(t)$ is the QPSK random communication signal, and $n(t)$ is AWGN noise. $\mathcal{H}_0$ represents the return when no target is present and $\mathcal{H}_1$ represents the return when a point target is present. In our application, we are more interested in the effect of QPSK interference rather than signal-dependent clutter. Thus, we assume that $P_x(f) >> P_h(f)$ in the frequency spectrum. Thus, we will assume $h(t)$ to be small in (5) and (6). Moreover, we conveniently transition to discrete-time signal model. Of course, we assume proper time sampling as dictated by the Nyquist sampling theorem. Thus, the discrete time detection hypotheses where we assume unit amplitude target for convenience and simulation purposes, i.e. $A = 1$, are given by

$$\mathcal{H}_0 : \mathbf{x} = \mathbf{q} + \mathbf{n} \tag{7}$$

$$\mathcal{H}_1 : \mathbf{x} = \mathbf{s} + \mathbf{q} + \mathbf{n}. \tag{8}$$

If the total interference is assumed to be Gaussian, then a generalized match filter detector is presented in [3]. The optimum detector for our problem should incorporate the fact that the QPSK interference is random (i.e. four phases) and thus the proper distribution. The proper distribution may also be a function of the relative received timing between the radar pulse and random interference symbol. In other words, the optimum detector is difficult to derive due to the addition of non-Gaussian interference to additive white Gaussian noise of the receiver. Here, we propose a suboptimal detector. Note that even though our friendly QPSK interference is not Gaussian, it is nonetheless a random signal whose autocorrelation function can easily be calculated. We temporarily assume that the total interference to be Gaussian such that we can use the generalized matched filter detector for correlated Gaussian noise. We use this as our sub-optimum detector in this work. For the purposes of generating interim performance curves (via theoretical calculations), we can calculate the detection performance since we have the correlation matrix of the QPSK signal $\mathbf{C}_q$ given by the matrix with the primary diagonal equal to σ_q^2. Each successive diagonal decreases by $\frac{1}{N}\sigma_q^2$ where N is the number of radar samples in a single QPSK symbol. Thus, the interference correlation matrix is given by

$$\mathbf{C}_q = \begin{bmatrix} \sigma_q^2 & \sigma_q^2(1-\frac{1}{N}) & \ldots & \sigma_q^2(1-\frac{N-1}{N}) & 0 \\ \sigma_q^2(1-\frac{1}{N}) & \sigma_q^2 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \sigma_q^2(1-\frac{1}{N}) & \sigma_q^2 \end{bmatrix}.$$

We let $\mathbf{C}$ be the correlation matrix of interference plus noise. Since the correlation matrix of AWGN noise is $\mathbf{C}_n = \sigma_n^2\mathbf{I}$, then the correlation matrix is given by

$$\mathbf{C} = \mathbf{C}_n + \mathbf{C}_q. \tag{9}$$

However, to produce more accurate performance curves we perform Monte Carlo simulations and compare these to the theoretical results. We start by looking at the *pdf* under each of the two hypotheses assuming the total interference to be correlated Gaussian process given by

$$p(\mathbf{x}|\mathcal{H}_0) = \frac{1}{\pi^N \det(\mathbf{C})} \exp[-\mathbf{x}^H\mathbf{C}^{-1}\mathbf{x}] \tag{10}$$

$$p(\mathbf{x}|\mathcal{H}_1) = \frac{1}{\pi^N \det(\mathbf{C})} \exp[-(\mathbf{x}-\mathbf{s})^H\mathbf{C}^{-1}(\mathbf{x}-\mathbf{s})] \tag{11}$$

where we decide $\mathcal{H}_1$ if a certain threshold is met, i.e. that a target is present if

$$\frac{p(\mathbf{x}|\mathcal{H}_1)}{p(\mathbf{x}|\mathcal{H}_0)} > \gamma \tag{12}$$

which is easily reduced to

$$\mathrm{Re}[-\tilde{s}^H\mathbf{C}^{-1}\mathbf{x}] > \gamma'. \tag{13}$$

It can be shown that the theoretical probability of detection P_D and probability of false alarm P_{FA} are given by

$$P_D = Q\left(\frac{\gamma' - \mathbf{s}^H C^{-1}\mathbf{s}}{\sqrt{\frac{\mathbf{s}^H C^{-1}\mathbf{s}}{2}}}\right) \tag{14}$$

$$P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\frac{\mathbf{s}^H\mathbf{C}^{-1}\mathbf{s}}{2}}}\right). \tag{15}$$

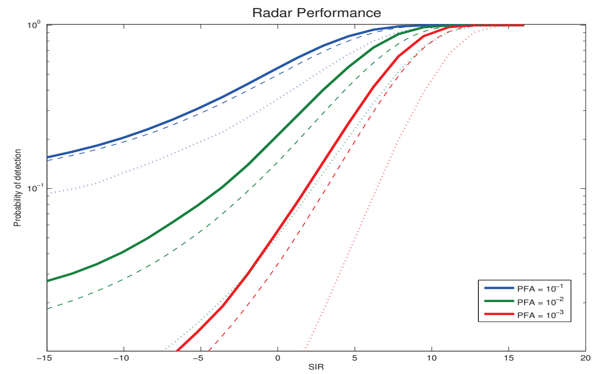

Fig. 3. Demonstrated probability of detection vs SIR for each selected probability of false alarm. Solid lines indicate the performance when the shaped spectrum pulse is used, dashed lines indicate theoretically calculated results and the dotted lines indicate demonstrated results using a classical pulse transmit waveform.

Solving equation (15) for γ' yields

$$\gamma' = \sqrt{\frac{\mathbf{s}^H\mathbf{C}^{-1}\mathbf{s}}{2}} Q^{-1}(P_{FA}) \tag{16}$$

and substituting (16) back into equation (14) yeilds

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{d^2}\right) \tag{17}$$

where the deflection coefficient is given by

$$d^2 = 2\mathbf{s}^H\mathbf{C}^{-1}\mathbf{s}. \tag{18}$$

In Fig. 3, we show the theoretical detection curves for the radar based on (17). Recall that (17) is only a theoretical approximation due to the fact that we assumed the total interference to be Gaussian. To account for the fact that the total interference is the sum of non-Gaussian QPSK random symbols and Gaussian receiver noise, we perform extensive Monte Carlo experiments to produce more accurate detection curves. The detection curves are close, albeit the theoretical curves are clearly pessimistic results compared to the actual Monte Carlo results. It should now be clear that the performance differences are attributed to the fact that the equations that are used assumed correlated Gaussian interference when in fact our QPSK interference is a random information signal. During Monte Carlo simulations, γ' was adjusted manually until the desired P_{FA} was attained.

Finally the detection performance of the optimum transmit waveform can be compared to the detection performance of a single pulsed radar with QPSK interference present. The goal of the optimum transmit waveform is to mitigate the effect of the interference (QPSK in this case). The performance curves in Fig. 3 illustrate the actual performance of the designed waveform is better than the performance predicted by the theoretically approximated equation. The performance of the designed waveform shows a marked improvement in detection performance over that of a pulsed radar. For a selected P_{FA} of 0.01 and a SIR of 3 dB we get an improvement in the probability of detection from 0.12 to 0.40 which shows a 3 to 1 improvement in detection.

IV. Existing Communication System

Another objective of the optimum transmit waveform design is to minimize the disruptive effect to the friendly communication system. In the case of a system employing QPSK modulation, our goal is to minimize the effect on the symbol error rate (SER) and/or probability of correct symbol detection P_s. The idea is not to make any system changes (software or hardware) to the legacy communication system, i.e. the communications system is unaware of the radar's beneficial waveform design. In other words, the legacy system remains intact while reducing radar disruption. This means the communications receiver uses the matched filter detector for QPSK without any additional signal processing. With the use of optimum transmit radar waveform, the result is the mitigation of effect on P_s performance.

In Fig. 4 we show the P_s of a legacy QPSK system. The redline indicates the performance of the system with no radar signal present. Here we have selected a SNR of 3 dB for the QPSK signal to AWGN noise. We then use a wideband pulse radar signal and simulate the probability of detecting the correct symbol with radar to QPSK power ratios ranging from -15 dB to +20 dB. From Fig. 4 we can see that the pulse has a disruptive effect on the communication system at about the -4 dB radar pulse to QPSK signal power ratio. We then show the performance of the un-altered QPSK detector when the spectral shaped transmit waveform from above is introduced. Here we see that this waveform does not begin to interfere with the communication detector until the 10 dB radar pulse to QPSK signal power ratio is reached. This is a marked improvement in the communication detector's ability to correctly demodulate the communication signal. In other words, this improvement comes from no adjustments to the legacy communication system but is simply a realized side effect of our optimum shaped radar pulse.

Fig. 5 shows the SER performance curves of a QPSK receiver when the radar signal is present. Here we can compare the SER performances when the shaped waveform signal is present and when traditional pulse radar transmit signal is present. To measure the effectiveness of the shaped transmit waveform in minimizing its effect on the communications systems, we also plot the ideal SER for the scenario where there is only white noise i.e. no radar present as the baseline performance. For the top panel of Fig. 5, the radar-to-noise ratio is 3 dB. In the middle panel it is 6 dB and finally the radar-to-noise ratio is 9 dB in the bottom panel. Baseline performance of QPSK modulation without a radar signal present is provided in each panel for comparison. As can be seen from the figure, the communication receiver works very well as if no radar transmit signal is present with even a 9 dB radar signal to noise ratio provided the radar is using the optimum shaped transmit pulse. In contrast, the communications receiver's performance is severely limited

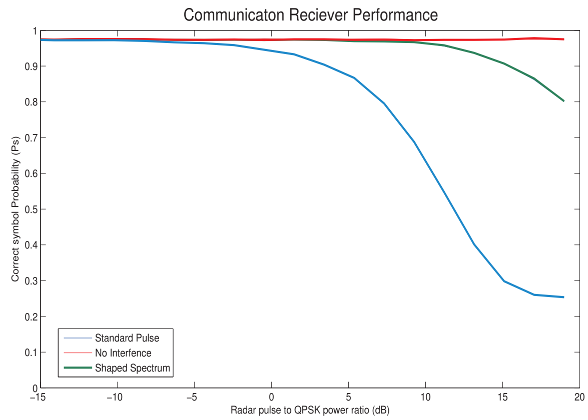

Fig. 4. Demonstrated detection of QPSK modulation via Monte Carlo simulations which are shown here under three scenarios. First we demonstrate the difficulty of detecting the symbol with a standard radar pulse present, then the improved communications receiver performance with the shaped pulse created by the radar. A baseline is presented of communication performance when no radar signal is present.

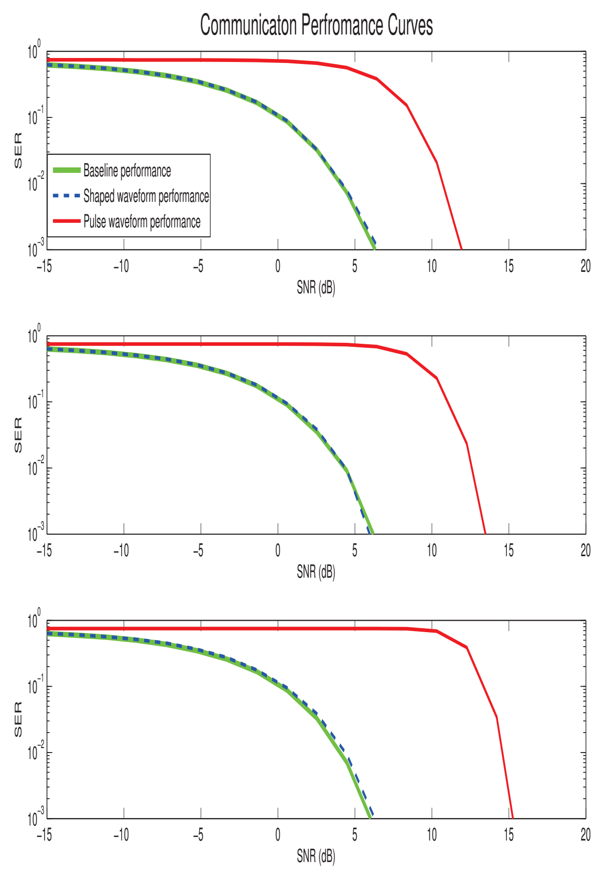

Fig. 5. Performance curves for QPSK receiver. Top panel: 3 dB radar-to-noise ratio. Middle panel: 6 dB radar-to-noise ratio. Bottom panel: 9 dB radar-to-noise power ratio.

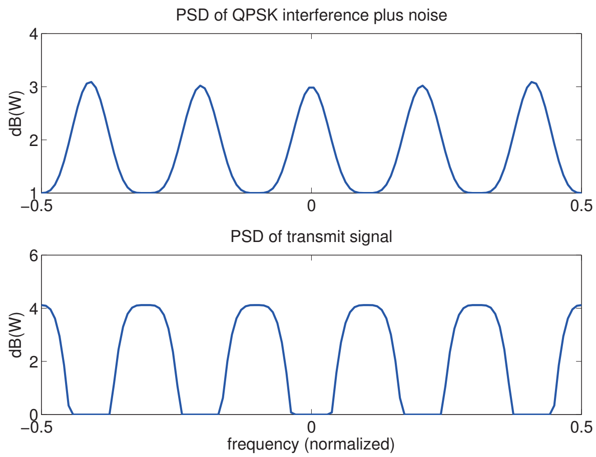

Fig. 6. Top Panel: PSD of multiple QPSK signals with bandwidth of 1/5 of the radar signal. Bottom panel: PSD of the resulting radar transmit signal.

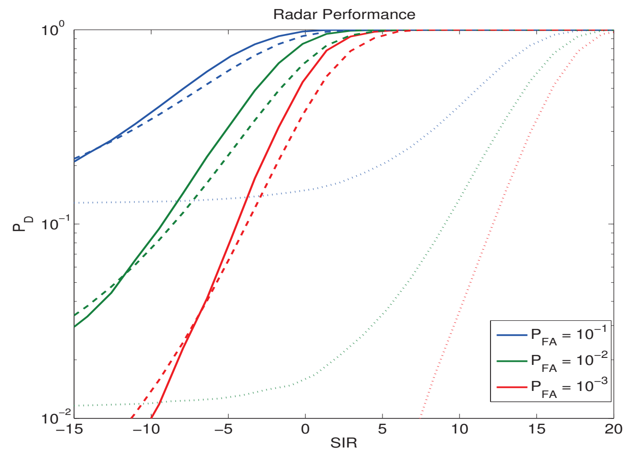

Fig. 7. Probability of detection versus SIR for each selected probability of false alarm for narrowband radar with five interferers. Dotted lines indicate performance due to traditional pulsed radar response, dashed lines are the theoretical predictions from (3.26) and the solid lines are the demonstrated performance of the optimum spectrum waveform.

even with a 3 dB radar signal to noise ratio if a traditional pulse transmit waveform is used.

V. Multiple Communication Systems

To further demonstrate the performance benefits of the waveform we now investigate the case when multiple communication signals are present in the portion of the spectrum available to the radar. In the top panel of Fig. 6 we see that there are five communication signals present in the portion of the spectrum available to the radar. In the bottom panel we see that the radar has placed its energy in the frequencies between these communication signals and avoided the center of each main lobe of the communication signals entirely. Thus, the corresponding radar's match filter will disregard the energy that is present from the communications systems. Also we see that since the energy is focused in smaller frequency bands, the peak power at each of these locations is now higher.

In Fig. 7 we can see the performance of the radar receiver once again. Notice that there is a dramatic increase in the radar's ability to detect targets over the traditional pulse radar. The pulse radar suffers dramatically due to the fact that the threshold must be set high enough to account for the five interfering communications signals.

Finally in Fig. 8 we see a sample of the performance of one of the five communications receivers. Here we notice that the performance deviates slightly from the theoretical performance of the QPSK demodulator but still shows a marked improvement over the case when the pulsed radar signal is present.

VI. Summary

In this paper we investigate a radar waveform design to be used in an active communication channel. We use the SNR-based 'waterfilling' technique. We then apply this technique to an active communication channel and create a sub-matched filter design that allows for recovery of a radar return. We have shown that this technique gives dramatic improvement

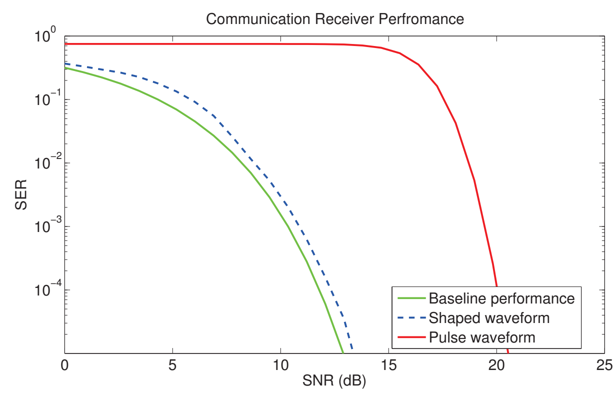

Fig. 8. Figure 22. Performance curves for QPSK receiver with 9dB interference to noise ratio for narrowband radar with five interferers.

over the use of a simple pulsed radar. Further we have also demonstrated that this new waveform minimally interferes with the receiver of the communication signal present in the environment. It is theorized that further work in this area could lead to a radar that is capable of sharing the spectrum with friendly communication systems that could reduce the strain on spectral resources.

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